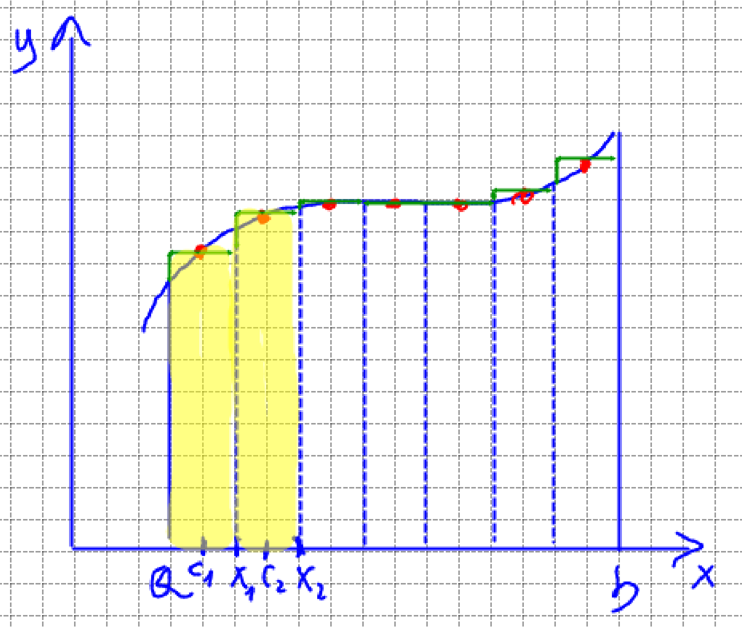
CALCOLO APPROSSIMATO DI UN INTEGRALE

Già sappiamo che $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=0}^{n-1} f(c_i)(x_{i+1}-x_i) =$

$= \lim_{n\to\infty} \sigma_n$ con $c_i \in [x_i ; x_{i+1}]$



Se divido $[a,b]$ in n parti uguali $h = \frac{b-a}{n} = x_{i+1} - x_i$

$c_1 = a + \frac{h}{2}$ $\quad c_2 = a + \frac{3}{2}h$ $\quad c_3 = a + \frac{5}{2}h$ ......

$$c_i = a + (2i-1)\frac{h}{2}$$

$$\sigma_n = h \sum_{i=1}^{n} f\left(a + (2i-1)\frac{h}{2}\right)$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} h \sum_{i=1}^{n} f\left(a + (2i-1)\frac{h}{2}\right)$$

ESEMPIO

$$\int_0^{\frac{\pi}{2}} \operatorname{sen} x\,dx = \left[-\cos x\right]_0^{\frac{\pi}{2}} = -\cos\frac{\pi}{2} + \cos 0 = 1$$

$I = \left[0 ; \frac{\pi}{2}\right]$ divido in tre parti uguali ho:

$I_1 = \left[0 ; \frac{\pi}{6}\right]$ $\quad I_2 = \left[\frac{\pi}{6} ; \frac{\pi}{3}\right]$ $\quad I_3 = \left[\frac{\pi}{3} ; \frac{\pi}{2}\right]$ $\quad h = \frac{\pi}{6}$

$c_1 = \frac{\pi}{12}$ $\quad c_2 = \frac{\pi}{4}$ $\quad c_3 = \frac{5}{12}\pi$

$\operatorname{sen} c_1 = 0{,}25882$ $\quad \operatorname{sen} c_2 = 0{,}70711$ $\quad \operatorname{sen} c_3 = 0{,}96593$

quindi

$$\int_0^{\frac{\pi}{2}} \operatorname{sen} x\,dx = h(\operatorname{sen} c_1 + \operatorname{sen} c_2 + \operatorname{sen} c_3) \cong 1{,}01152$$