ESEMPI INTEGRALI IMPROPRI

$$f(x)=\frac{1}{x} \qquad (0;1] \qquad D_f=\mathbb{R}-\{0\}=(-\infty;0)\cup(0;+\infty)$$

$$\lim_{b\to 0^+}\int_b^1 f(x)\,dx=\lim_{b\to 0^+}\int_b^1 \frac{1}{x}\,dx=\lim_{b\to 0^+}\Big[\ln|x|\Big]_b^1=$$

$$=\lim_{b\to 0^+}\left(\ln 1-\ln b\right)=-\left(\ln 0^+\right)=-(-\infty)=+\infty$$

ESEMPIO

$$f(x)=(x^2-1)e^{-x} \qquad \text{calcolare } \int_1^{+\infty}(x^2-1)e^{-x}\,dx$$

$$D_f=\mathbb{R}$$

$$\text{sia } b>1 \text{ e } F(b)=\int_1^b (x^2-1)e^{-x}\,dx$$

$$\lim_{b\to+\infty}F(b)=\lim_{b\to+\infty}\int_1^b (x^2-1)e^{-x}\,dx$$

$$F(b)=\int_1^b \frac{x^2-1}{e^x}\,dx=\int_1^b\frac{x^2}{e^x}\,dx-\int_1^b\frac{1}{e^x}\,dx=$$

$$\frac{1}{e^x}=e^{-x}$$

$$=\int_1^b\frac{x^2}{e^x}\,dx+\int_1^b e^{-x}\,d(-x)=$$

$$=\int_1^b\frac{x^2}{e^x}\,dx+\Big[e^{-x}\Big]_1^b=$$

$$=\left[-x^2e^{-x}-\int -2xe^{-x}\,dx\right]_1^b+\left[e^{-b}-e^{-1}\right]=$$

$$=\left[-x^2e^{-x}+2\int xe^{-x}\,dx\right]_1^b+\left[e^{-b}-\frac{1}{e}\right]=$$

$$=\left[-x^2e^{-x}-2xe^{-x}-2e^{-x}\right]_1^b+\left[\frac{1}{e^b}-\frac{1}{e}\right]=$$

$$=-b^2e^{-b}-2be^{-b}-2e^{-b}+e^{-1}+2e^{-1}+2e^{-1}+\frac{1}{e^b}-\frac{1}{e}=$$

$$=-\frac{b^2}{e^b}-\frac{2b}{e^b}-\frac{2}{e^b}+\cancel{\frac{1}{e}}+\frac{2}{e}+\frac{2}{e}+\frac{1}{e^b}-\cancel{\frac{1}{e}}=$$

$$=-\frac{b^2}{e^b}-\frac{2b}{e^b}-\frac{1}{e^b}+\frac{4}{e}=\frac{-1}{e^b}(b+1)^2+\frac{4}{e}$$

$$\lim_{b\to+\infty} F(b) = \lim_{b\to+\infty} \frac{-1}{e^{b}}(b+1)^2 + \frac{4}{e} = \frac{4}{e}$$