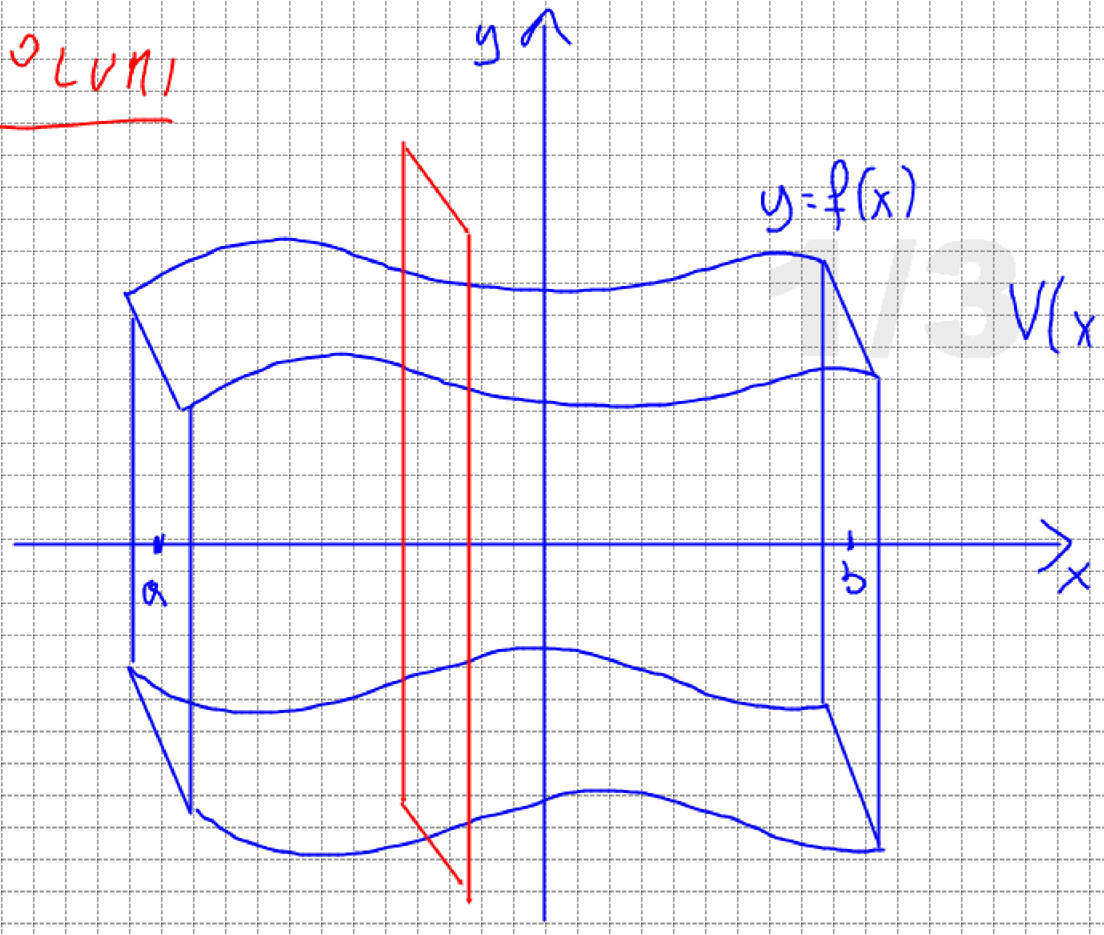
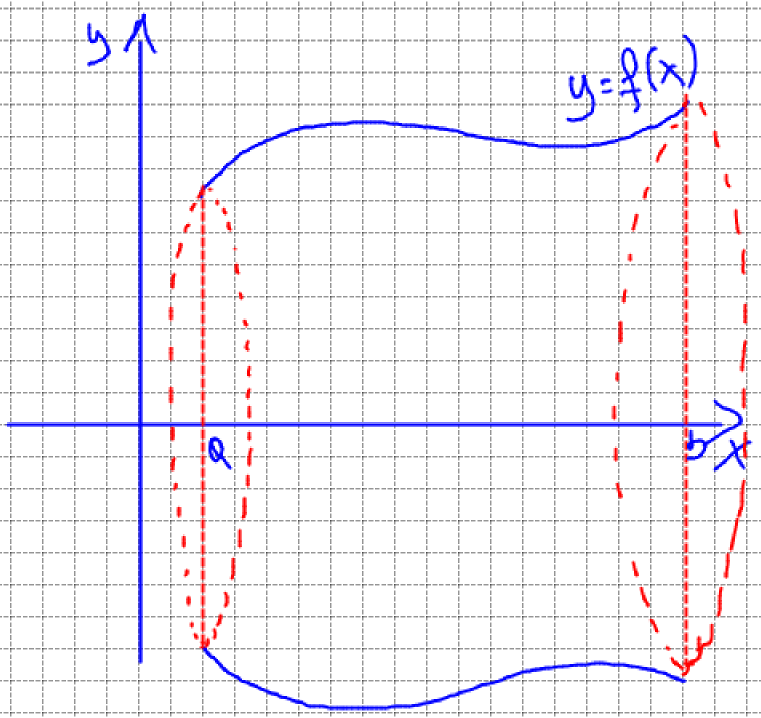
# VOLUMI



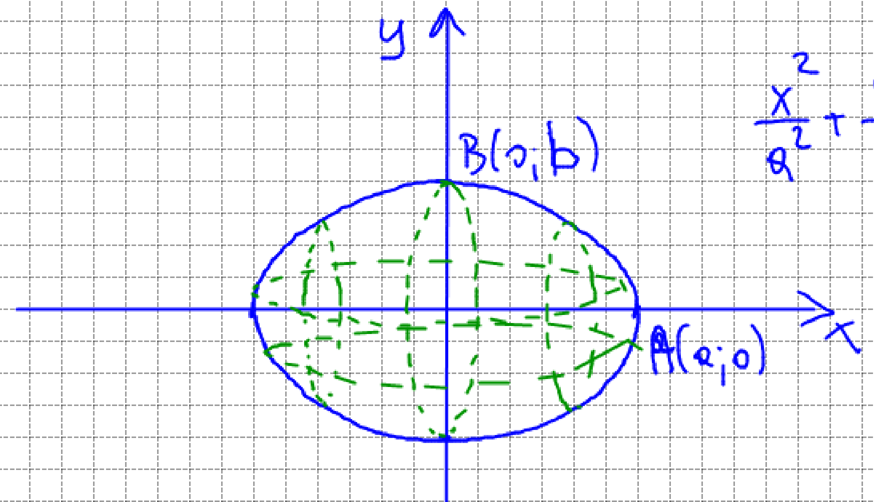

$$V(x)=\int_a^b S(x)\,dx$$

## VOLUME DEI SOLIDI DI ROTAZIONE

$$V=\pi\int_a^b f^2(x)\,dx$$




## VOLUME ELLISSOIDE




$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2} \qquad y^2=b^2\left(1-\frac{x^2}{a^2}\right) \qquad y=\pm b\sqrt{1-\frac{x^2}{a^2}}$$

$$y=b\sqrt{1-\frac{x^2}{a^2}} \qquad f(x)$$

$$V=\pi\int_{-a}^{a} f^2(x)\,dx=2\pi\int_0^a f^2(x)\,dx=$$

$$=2\pi b^2\int_0^a \cancel{\sqrt{1-\frac{x^2}{a^2}}}^{\,2}\,dx=\frac{2\pi b^2}{a^2}\int_0^a (a^2-x^2)\,dx=$$

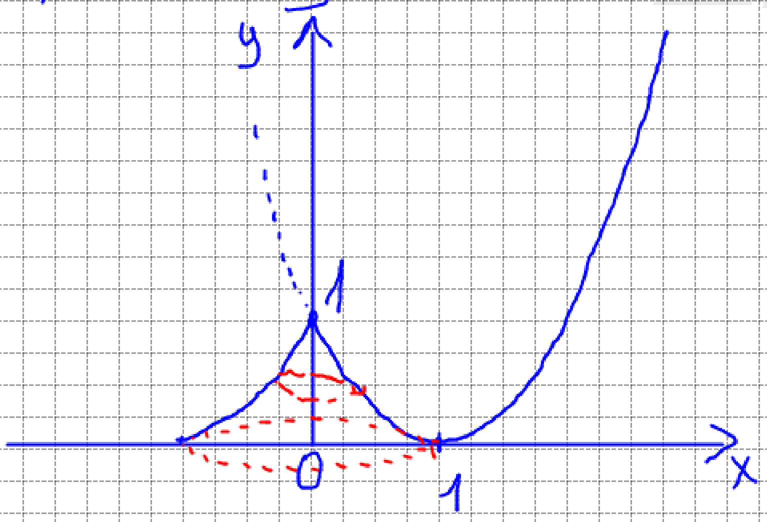
$$=\frac{2\pi b^2}{a^2}\left[a^2x-\frac{x^3}{3}\right]_0^a=$$

$$=\frac{2\pi b^2}{a^2}\left[a^3-\frac{a^3}{3}\right]=\frac{2\pi b^2}{\cancel{a^2}}\cdot\frac{2}{3}a^{\cancel{3}}=\frac{4}{3}\pi a b^2$$

$$\frac{4}{3}\pi a^2 b$$

## ESERCIZIO

Data la parabola $y = x^2 - 2x + 1$




$x = 1 \pm \sqrt{y}$ tra 0 e 1 ho la curva $1 - \sqrt{y} = x$

$$V = \pi \int_0^1 \left[1 - \sqrt{y}\right]^2 dy = \pi \int_0^1 \left(1 + y - 2\sqrt{y}\right) dy =$$

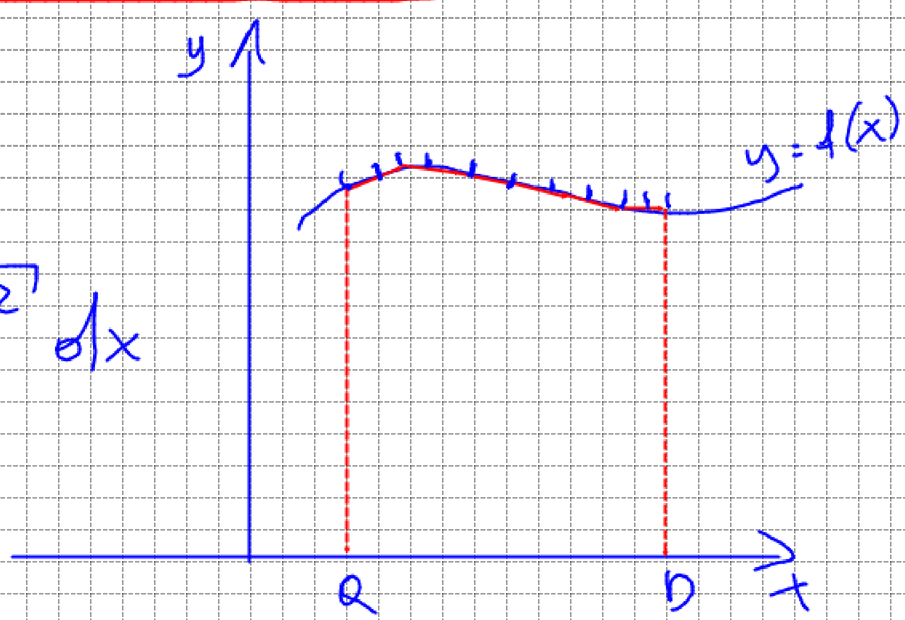
$$= \pi \left[ y + \frac{y^2}{2} - 2\,\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^1 =$$

$$= \pi \left(1 + \frac{1}{2} - \frac{4}{3}\right) = \frac{\pi}{6}$$

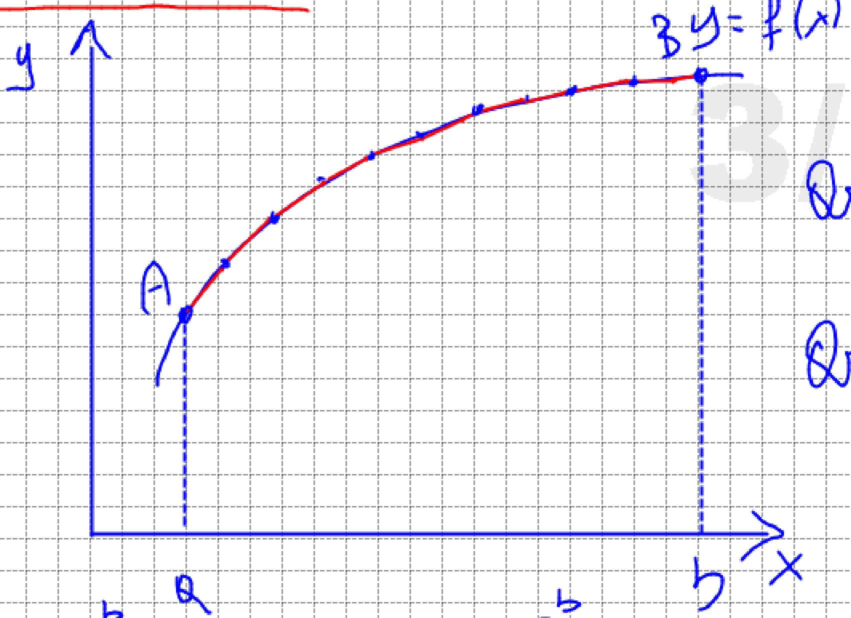
## LUNGHEZZA ARCO DI CURVA

$y = f(x)$

$$l = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$



# TEOREMA DI GULDINO




$$Area_y = 2\pi |x_G| L$$

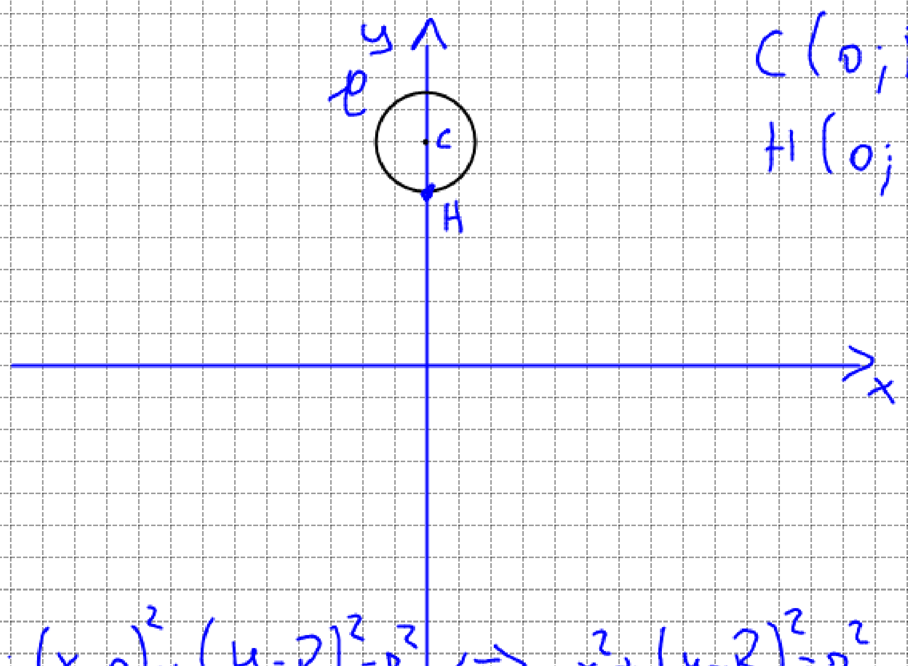
$$Area_x = 2\pi |y_G| L$$

$$m \int_a^b \sqrt{1+f'(x)^2}\, dx \le \int_a^b f(x)\sqrt{1+f'(x)^2}\, dx \le M \int_a^b \sqrt{1+f'(x)^2}\, dx$$

con $\int_a^b \sqrt{1+f'(x)^2}\, dx = L$

$$m \le \underbrace{\frac{\int_a^b f(x)\sqrt{1+f'(x)^2}\, dx}{L}}_{y_G} \le M$$

# AREA DEL TORO




$C(0; R)$

$H(0; R-r)$

$\overline{CH} = r$

$$\ell : (x-0)^2 + (y-R)^2 = r^2 \iff x^2 + (y-R)^2 = r^2$$

$$x^2 + y^2 + R^2 - 2Ry = r^2$$

$$y_G = R$$

$$Area_T = 2\pi |y_G| \overbrace{L}^{\text{lunghezza circonferenza}} = 2\pi R \cdot 2\pi r = 4\pi^2 R r$$