

SOSTITUZIONI CON FUNZIONI GONIOMETRICHE

$$\bullet \int \sqrt{a^2 - x^2} dx = \quad \text{poniamo } x = a \cos t$$

$$dx = -a \sin t dt$$

$$= \int \sqrt{a^2 - a^2 \cos^2 t} (-a \sin t dt) =$$

$$= \int \sqrt{a^2 (1 - \cos^2 t)} (-a \sin t dt) = \int \sqrt{a^2 \sin^2 t} (-a \sin t dt)$$

$$= \int a \sin t (-a \sin t dt) = -\frac{1}{2} \int \sin^2 t dt = (*)$$

$$\underline{\cos 2t} = \frac{\cos^2 t - \sin^2 t}{1 - \sin^2 t} = \underline{1 - 2 \sin^2 t}$$

$$\sin^2 t = \frac{1 - \cos 2t}{2}$$

$$(*) = -a^2 \int \frac{1 - \cos 2t}{2} dt = -\frac{a^2}{2} \left[\int 1 dt - \int \cos 2t dt \right]$$

$$= -\frac{a^2}{2} \left[\frac{1}{2} t - \frac{1}{4} \sin 2t + k_1 \right] = -\frac{1}{2} a^2 t + \frac{1}{4} a^2 \sin 2t + k$$