$\boxed{(3)}$ $\Delta < 0$ $\quad \displaystyle\int \frac{1}{ax^2+bx+c}\,dx \quad \Delta < 0$.

$ax^2+bx+c = (x+m)^2 + n^2$

ESEMPIO

$$\int \frac{1}{x^2+x+1}\,dx =$$

$x^2+x+1 = x^2 + 2\cdot\frac{1}{2}\cdot x + 1 + \frac{1}{4} - \frac{1}{4} =$

$= \left(x^2 + 2\frac{1}{2}x + \frac{1}{4}\right) + \frac{3}{4} =$

$= \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$

$$= \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}\,dx =$$

$$= \int \frac{1}{\frac{3}{4}\left[\frac{\left(x+\frac{1}{2}\right)^2}{\frac{3}{4}} + 1\right]}\,dx =$$

$$= \frac{4}{3}\int \frac{1}{\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)^2 + 1}\,dx = \frac{4}{3}\int \frac{1}{\left[\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right]^2 + 1}\,dx =$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{4}{3}\int \frac{1}{\Big[\underbrace{\boxed{\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)}}_{X}\Big]^2 + 1}\,d\Big(\overbrace{\boxed{\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)}}^{X}\Big) =$$

$$= \frac{2\sqrt{3}}{3}\int \frac{1}{X^2+1}\,dX = \frac{2\sqrt{3}}{3}\operatorname{arctg}\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right) + c.$$

ESEMPIO

$$\int \frac{3x+1}{x^2+9}\,dx = \int \frac{3x}{x^2+9}\,dx + \int \frac{1}{x^2+9}\,dx =$$

$$= \frac{3}{2}\int \frac{\overset{f'(x)}{2x}}{\underset{f(x)}{x^2+9}}\,dx + \int \frac{1}{x^2+9}\,dx =$$

$\frac{1}{x^2+9} \rightarrow \frac{1}{9\left(\frac{x^2}{9}+1\right)}$

$$= \frac{3}{2}\ln\left|x^2+9\right| + c_1 + \frac{1}{9}\cdot 3\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,d\left(\frac{x}{3}\right) =$$

$$= \frac{3}{2}\ln\left|x^2+9\right| + c_1 + \frac{1}{3}\cdot\operatorname{arctg}\left(\frac{x}{3}\right) + c_2$$

$ax^2+bx+c=0$ cu $\Delta<0$

$x_{1,2}=m\pm ni \qquad i=\sqrt{-1} \Leftrightarrow i^2=-1$

$$a(x-m+ni)(x-m-ni)=0$$

$$a\left[(x-m)^2-(ni)^2\right]=0$$

$$a\left[(x-m)^2+n^2\right]=0$$

$x^2+x+1=0 \qquad x_{1,2}=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{-3}}{2}=$

$$=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$$

$$\left(\left(x+\frac{1}{2}\right)-\frac{\sqrt{3}}{2}i\right)\left(\left(x+\frac{1}{2}\right)+\frac{\sqrt{3}}{2}i\right)=0$$

$\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}i\right)^2=0 \qquad \left(x+\frac{1}{2}\right)^2+\frac{3}{4}=0$

$\frac{3}{4}i^2=-\frac{3}{4} \qquad i^2=-1 \qquad i=\sqrt{-1}$

INTEGRALI CON MODULI

Data la funzione $y = f(x)$

- $\int f(x)\,dx$ rappresenta la famiglia delle primitive di $f(x)$
- $\int |f(x)|\,dx$ rappresenta la famiglia delle primitive di $|f(x)|$

$$|f(x)| \begin{cases} = f(x) & \text{se } f(x) \geq 0 \\ = -f(x) & \text{se } f(x) < 0 \end{cases}$$

ESEMPIO

$$\int |x|\,dx = \qquad |x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

se $x \geq 0$

$$\int |x|\,dx = \int x\,dx = \frac{1}{2}x^2 + c$$

se $x < 0$

$$\int |x|\,dx = \int -x\,dx = -\frac{1}{2}x^2 + c$$

$$\int |x|\,dx = c + \begin{cases} \frac{1}{2}x^2 & \text{se } x \geq 0 \\ -\frac{1}{2}x^2 & \text{se } x < 0 \end{cases}$$

$$\int |x|\,dx = \frac{1}{2}x|x| + c$$