

# INTEGRALE INDEFINITO

## PRIMITIVA DI UNA FUNZIONE

- Sia  $y = f(x)$ ,  $y = F(x) + K$   $\forall K \in \mathbb{R}$  è la famiglia di primitive di  $y = f(x)$  se  $F'(x) = f(x)$ .

- Si dice integrale indefinito di  $y = f(x)$ :

*funzione integranda*  $\leftarrow \int f(x) dx \rightarrow$  *integrazione indefinita*

La Totalità delle primitive della funzione  $y = f(x)$

### ESEMPIO

- $f(x) = 3$

$$\int 3 dx = 3x + K \quad K \in \mathbb{R} \quad D(3x + K) = 3$$

- $f(x) = x$

$$\int x dx = \frac{1}{2}x^2 + K \quad K \in \mathbb{R} \quad D\left(\frac{1}{2}x^2 + K\right) = \frac{2}{2}x = x$$

- $f(x) = \sin x$

$$\int \sin x dx = -\cos x + K \quad K \in \mathbb{R} \quad D(-\cos x + K) = -(-\sin x) = \sin x$$

- $f(x) = \cos x$

$$\int \cos x dx = \sin x + K \quad K \in \mathbb{R} \quad D(\sin x + K) = \cos x$$

- $f(x) = e^x$

$$\int e^x dx = e^x + K \quad K \in \mathbb{R} \quad D(e^x + K) = e^x$$

- $f(x) = \frac{1}{|x|}$

$$\int \frac{1}{|x|} dx = \ln|x| + K \quad K \in \mathbb{R} \quad D(\ln|x| + K) = \frac{1}{|x|}$$

## PROPRIETÀ DELL'INTEGRALE INDEFINITO

$$1) \int k f(x) dx = k \int f(x) dx \quad k \in \mathbb{R}$$

$$2) \int [f_1(x) + f_2(x) + \dots + f_n(x)] dx = \int f_1(x) dx + \int f_2(x) dx + \dots + \int f_n(x) dx$$

$$\bullet \int (x + x^2 + x^3) dx = \int x dx + \int x^2 dx + \int x^3 dx = \frac{1}{2} x^2 + \frac{1}{3} x^3 + \frac{1}{4} x^4 + C$$

## INTEGRALI IMMEDIATI

$$\bullet \int x^m dx = \frac{1}{m+1} x^{m+1} + k \quad k \in \mathbb{R} \quad (m \neq -1)$$

$$\bullet \int \frac{1}{x} dx = \ln|x| + k \quad k \in \mathbb{R}$$

$$\bullet \int \sin x dx = -\cos x + k \quad k \in \mathbb{R}$$

$$\bullet \int \cos x dx = \sin x + k \quad k \in \mathbb{R}$$

$$\bullet \int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + k \quad k \in \mathbb{R}$$

$$\bullet \int \frac{1}{\sin^2 x} dx = -\operatorname{ctg} x + k \quad k \in \mathbb{R}$$

$$\bullet \int e^x dx = e^x + k, k \in \mathbb{R}$$

$$\bullet \int a^x dx = a^x \log_a e + k, k \in \mathbb{R}$$

$$\bullet \int \frac{1}{\sqrt{1-x^2}} dx = \operatorname{arcsin} x + k$$

$$\bullet \int \frac{1}{1+x^2} dx = \operatorname{arctg} x + k$$

## REGOLE DI INTEGRAZIONE DELLE FUNZIONI COMPOSITE

$$\int [f(x)]^\alpha \underbrace{f'(x)}_{d(f(x))} dx = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + k \quad k \in \mathbb{R} \quad \alpha \neq -1$$

### ESEMPIO

$$\int (3x)^5 \underbrace{3 dx}_{d(3x)} = \frac{(3x)^{5+1}}{5+1} + k \quad k \in \mathbb{R}$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + k$$

$$\int f'(x) \operatorname{sen} f(x) dx = -\cos f(x) + k$$

$$\int f'(x) \cos f(x) dx = \operatorname{sen} f(x) + k$$

$$\int \frac{f'(x)}{\cos^2 f(x)} dx = \operatorname{Tg}(f(x)) + k$$

$$\int \frac{f'(x)}{\operatorname{sen}^2 f(x)} dx = -\operatorname{ctg} f(x) + k$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + k$$

$$\int f'(x) a^{f(x)} dx = a^{f(x)} \frac{1}{\ln a} + k$$

$$\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}} dx = \operatorname{arcsen} f(x) + k$$

$$\int \frac{f'(x)}{1+[f(x)]^2} dx = \operatorname{arctg} f(x) + k$$

### ESEMPLI

$$\int -\operatorname{sen} x \cos^4 x dx = -\frac{[\cos x]^5}{5} + k \quad k \in \mathbb{R}$$

$$\int 2 \sqrt{2x+1} dx = \int (2x+1)^{\frac{1}{2}} d(2x+1) = \frac{(2x+1)^{\frac{1}{2}+1}}{\frac{3}{2}} + k$$

### ESERCIZI

$$\int \operatorname{Tg} x dx; \quad \int \frac{1}{2x+1} dx; \quad \int \cos 2x dx; \quad \int \frac{1}{\cos^2 3x} dx;$$

$$\int \frac{1}{\operatorname{sen}^2(3-x)} dx$$