ESERCIZIO N° 1

$$f(x) = \begin{cases} 2e^{\frac{ax+b}{x-c}} & x \geq 0;\ x \neq c \quad c \in \mathbb{R}^+ \\ \frac{\operatorname{sen} 2x}{x} & x < 0 \end{cases}$$

$a, b, c,$ in modo tale che:

- $f(x)$ continua in $x=0 \Rightarrow$ $\begin{cases} 2e^{\frac{b}{-c}} = \lim\limits_{x \to 0^-} \frac{\operatorname{sen} 2x}{x} \\ \lim\limits_{x \to +\infty} 2e^{\frac{ax+b}{x-c}} = 2e \\ \lim\limits_{x \to 3^-} 2e^{\frac{ax+b}{x-c}} = 0 \end{cases}$
- $\lim\limits_{x \to +\infty} f(x) = 2e$
- $\lim\limits_{x \to 3^-} f(x) = 0$

$$\begin{cases} 2e^{-\frac{b}{c}} = 2 \\ a = 1 \\ 3 - c \to 0 \end{cases} \qquad \begin{cases} e^{-\frac{b}{c}} = 1 \\ a = 1 \\ c \to 3 \end{cases}$$

$$\begin{cases} -\frac{b}{c} = 0 \\ a = 1 \\ c = 3 \end{cases} \qquad \begin{cases} b = 0 \\ a = 1 \\ 3 - c = 0 \Rightarrow c = 3 \end{cases}$$

$$f(x) = \begin{cases} 2e^{\frac{x}{x-3}} & x \geq 0 \quad x \neq 3 \\ \frac{\operatorname{sen} 2x}{x} & x < 0 \end{cases}$$

b) $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} 2e^{\frac{x}{x-3}} = +\infty$

$$\lim_{x \to -\infty} F(x) = \lim_{x \to -\infty} \frac{\operatorname{sen} 2x}{x} = 0$$

c) $\lim\limits_{x \to -\infty} x F(x) = \lim\limits_{x \to -\infty} \cancel{x} \frac{\operatorname{sen} 2x}{\cancel{x}} = \nexists$ limite

$$\lim_{x \to 0^-} \frac{f(x)}{x} = \lim_{x \to 0^-} \frac{\operatorname{sen} 2x}{x} \cdot \frac{1}{x} =$$

$$\lim_{x \to 0^-} \frac{\operatorname{sen} 2x}{x} \cdot \lim_{x \to 0^-} \frac{1}{x} = -\infty$$

Sia $y = f(x)$ una funzione derivabile almeno n volte in $(a;b)$

E sia che:

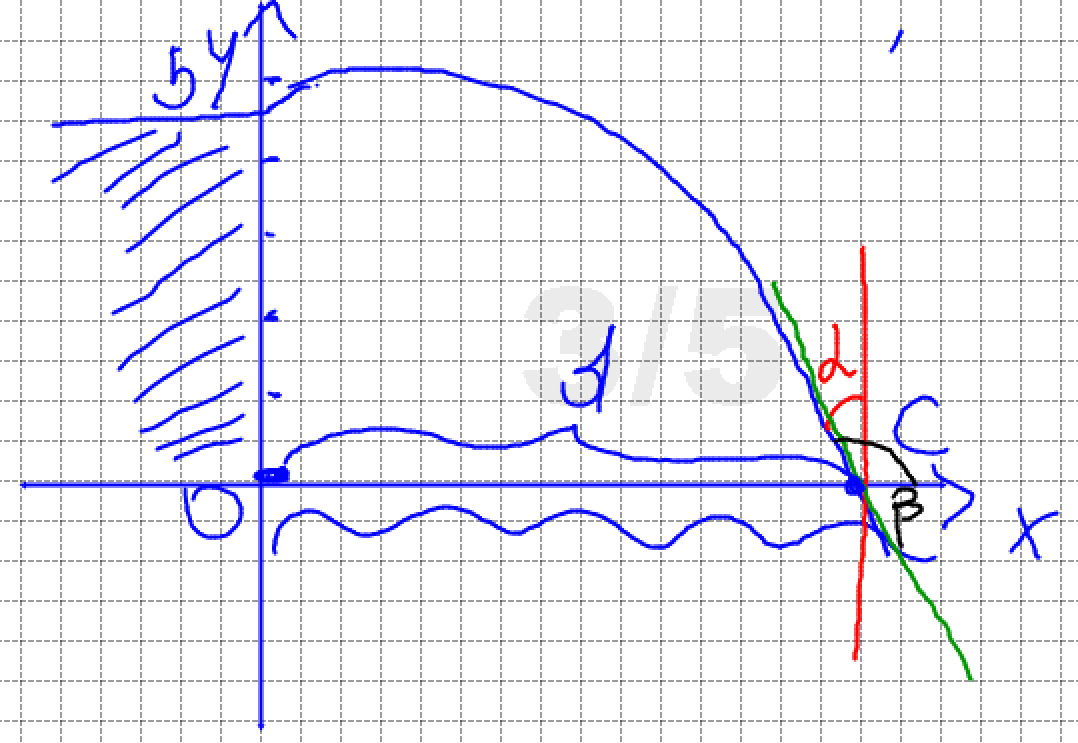
$f'(x_0), f''(x_0), f'''(x_0), \ldots\ldots f^{n-1}(x_0) = 0$ con $f^n(x) \neq 0$

Se n è PARI il punto $(x_0; F(x_0))$ è MASSIMO O MINIMO di $f(x)$ a seconda che $f^n(x_0)$ sia minore o maggiore di zero.

Se n è DISPARI il punto $(x_0; f(x_0))$ è un PUNTO DI FLESSO A TANGENTE ORIZZONTALE ascendente

Problema

$h = 5$ m



$$y = -\frac{1+m^2}{10} x^2 + mx + 5 \qquad m \in \mathbb{R}$$

a) $\operatorname{tg}\alpha = ?$ $\qquad 0 < \alpha < \frac{\pi}{2}$

$$0 < \operatorname{tg}\alpha < \frac{\sqrt{2}}{2}$$

trovo le coordinate di $C(x_c; 0)$

$$\begin{cases} y = 0 \\ -\frac{1+m^2}{10} \cdot x^2 + mx + 5 = 0 \end{cases}$$

$$\begin{cases} y = 0 \\ (-1-m^2)x^2 + 10mx + 50 = 0 \end{cases}$$

$$x_{1,2} = \frac{-5m \pm \sqrt{25m^2 + 50 + 50m^2}}{-1-m^2}$$

$$x_{1,2} = \frac{-5m \pm \sqrt{75m^2 + 50}}{-1-m^2}$$

$$x_{1,2} = \frac{-5m \pm 5\sqrt{3m^2+2}}{-1-m^2}$$

$$x_C = \frac{-5m + 5\sqrt{3m^2+2}}{-1-m^2}$$

$$f'(x_c) = y'\Big|_{x=x_c} = \left(-\frac{1+m^2}{5}x + m\right)\Bigg|_{x=x_c} =$$

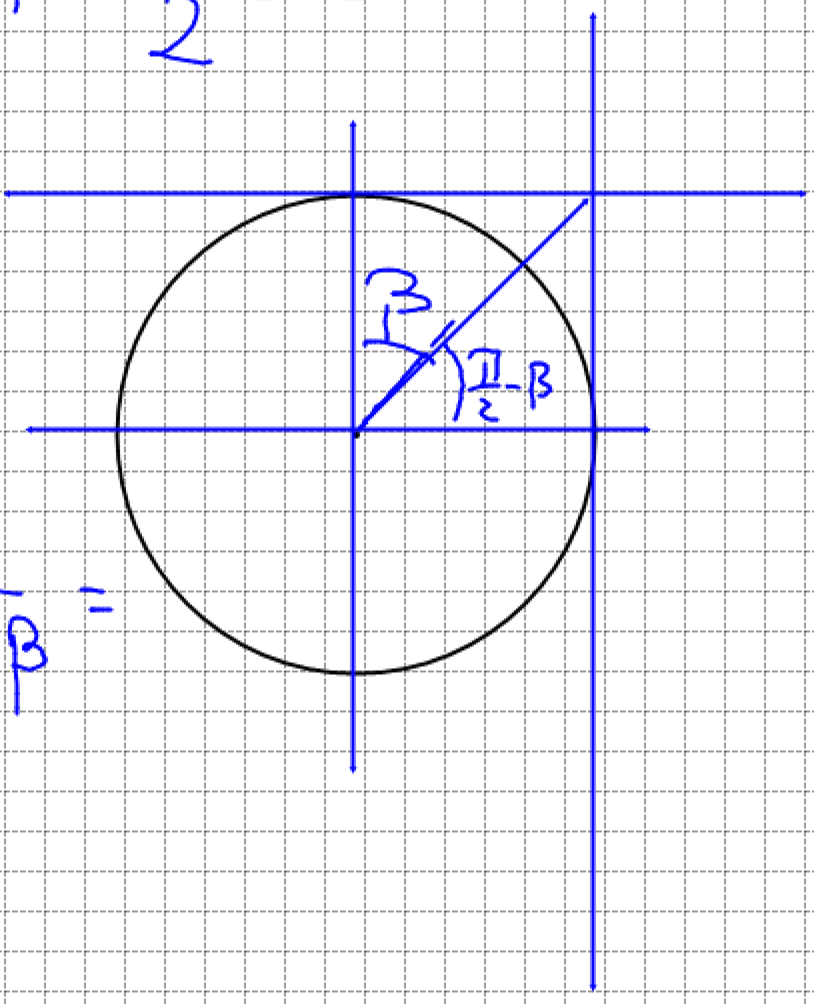
$$= -\frac{\cancel{1+m^2}}{\cancel{5}}\left(\frac{-\cancel{5}}{-1}\right)\left(\frac{m+\sqrt{3m^2+2}}{\cancel{1+m^2}}\right) + m =$$

$$= -\cancel{m} - \sqrt{3m^2+2} + \cancel{m} = -\sqrt{3m^2+2}$$

$$\operatorname{tg}\beta = -\sqrt{3m^2+2}$$

$$\beta = \frac{\pi}{2} + \alpha \Rightarrow \alpha = \beta - \frac{\pi}{2}$$

$$\operatorname{tg}\alpha = \operatorname{tg}\left(\beta - \frac{\pi}{2}\right) =$$



$$= -\operatorname{tg}\left(\frac{\pi}{2} - \beta\right)$$

$$= -\operatorname{ctg}\beta = -\frac{1}{\operatorname{tg}\beta} =$$

$$= \frac{1}{\sqrt{3m^2+2}}$$

$$\operatorname{tg}\alpha = \frac{1}{\sqrt{3m^2+2}}$$

- Dimostrare $0 < \operatorname{tg}\alpha \leq \frac{\sqrt{2}}{2}$

$$D_{\operatorname{tg}\alpha} = \left\{ m \in \mathbb{R} \,/\, \begin{cases} \sqrt{3m^2+2} \neq 0 \\ 3m^2+2 > 0 \end{cases} \right\} = (-\infty; +\infty)$$

- $\operatorname{tg}\alpha$ è simmetrica rispetto all'asse y

$$f(m) = f(-m) \quad (\text{PARI})$$

- $\operatorname{tg}\alpha$ è sempre positiva

- $$\operatorname{tg}\alpha \Big|_{m=0} = \left(\frac{1}{\sqrt{3\cdot(0)^2+2}}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

- $$\operatorname{tg}\alpha \Big|_{m\to+\infty} = \lim_{m\to+\infty} \frac{1}{\sqrt{3m^2+2}} = 0$$

$$0 < \operatorname{tg}\alpha \leq \frac{\sqrt{2}}{2}$$

$$0 < \frac{1}{\sqrt{3m^2+2}} \leq \frac{\sqrt{2}}{2}$$

b) $m > 0$ $\alpha = 30°$

$d = ?$ tra la base della piattaforma e il punto C.