FUNZIONI RAZIONALI FRATTE (LIMITI $x \to \infty$)

$$\lim_{x\to\infty} \frac{a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \dots + b_1 x + b_0} =$$

$g(N) = n \qquad g(D) = m$

$$= \lim_{x\to\infty} \frac{x^n\left(a_n + \frac{a_{n-1}}{x} + \dots + \frac{a_1}{x^{n-1}} + \frac{a_0}{x^n}\right)}{x^m\left(b_m + \frac{b_{m-1}}{x} + \dots + \frac{b_1}{x^{m-1}} + \frac{b_0}{x^m}\right)} =$$

$$= \begin{cases} n = m & = \dfrac{a_n}{b_m} \\ n > m & = \infty \\ n < m & = \dfrac{1}{\infty} \to 0 \end{cases}$$

ESEMPIO

$$\lim_{x\to\infty} \frac{4x^4+5}{3x^4+2x+1} = \lim_{x\to\infty} \frac{\cancel{x^4}\left(4 + \frac{5}{x^4}\right)}{\cancel{x^4}\left(3 + \frac{2}{x^3} + \frac{1}{x^4}\right)} = \frac{4}{3}$$

$y = \frac{4}{3}$ Asintoto orizzontale.

ESEMPIO

$$\lim_{x\to\infty} \frac{3x^3+2x}{4x} = \lim_{x\to\infty} \frac{\cancel{x^3}^{\,2}\left(3 + \frac{2}{x^2}\right)}{4\cancel{x}} = \infty$$

ESEMPIO

$$\lim_{x\to\infty} \frac{5}{4x^2+3} = 0$$

ESEMPIO

$$\lim_{x\to-\infty} \frac{3x^4-5x+8}{x^5-7} = (\bullet)$$

$$\left[\frac{+\infty+\infty+8}{-\infty-7} \to \frac{+\infty}{-\infty} \to F.I. \right]$$

$$(\bullet) = \lim_{x\to-\infty} \frac{\cancel{x^4}\left(3 - \frac{5}{x^3} + \frac{8}{x^4}\right)}{x^{\cancel{5}}\left(1 - \frac{7}{x^5}\right)} = 0^-$$

$y = 0$ asintoto orizzontale (inferiore)

LIMITI FONDAMENTALI

LIMITE INFINITO ALL'INFINITO ($n \in \mathbb{N}_0$)	LIMITE FINITO ALL'INFINITO ($n \in \mathbb{N}_0$)	LIMITE INFINITO IN UN PUNTO ($n \in \mathbb{N}_0$)
$\lim_{x \to \pm\infty} x^{2n} = +\infty$	$\lim_{x \to \pm\infty} \frac{1}{x^n} = 0$	$\lim_{x \to 0} \frac{1}{x^{2n}} = +\infty$
$\lim_{x \to +\infty} x^{2n+1} = +\infty$	$\lim_{x \to +\infty} \frac{1}{\sqrt[2n]{x}} = 0$	$\lim_{x \to 0^+} \frac{1}{x^{2n+1}} = +\infty$
$\lim_{x \to -\infty} x^{2n+1} = -\infty$	$\lim_{x \to \pm\infty} \frac{1}{\sqrt[2n+1]{x}} = 0$	$\lim_{x \to 0^-} \frac{1}{x^{2n+1}} = -\infty$
$\lim_{x \to +\infty} e^x = +\infty$	$\lim_{x \to -\infty} e^x = 0$	$\lim_{x \to 0^+} \ln x = -\infty$
$\lim_{x \to +\infty} \ln x = +\infty$	$\lim_{x \to -\infty} a^x = 0 \ (a>1)$	$\lim_{x \to \frac{\pi}{2}^-} \operatorname{tg} x = +\infty$
$\lim_{x \to +\infty} \log_a x = -\infty \ (0<a<1)$	$\lim_{x \to +\infty} a^x = 0 \ (0<a<1)$	$\lim_{x \to \frac{\pi}{2}^+} \operatorname{tg} x = -\infty$
$\lim_{x \to +\infty} \log_a x = +\infty \ (a>1)$	$\lim_{x \to +\infty} \operatorname{arctg} x = \frac{\pi}{2}$	$\lim_{x \to 0^+} \operatorname{ctg} x = +\infty$
$\lim_{x \to \pm\infty} \cosh x = +\infty$	$\lim_{x \to -\infty} \operatorname{arctg} x = -\frac{\pi}{2}$	$\lim_{x \to 0^-} \operatorname{ctg} x = -\infty$
$\lim_{x \to +\infty} \operatorname{senh} x = +\infty$	$\lim_{x \to +\infty} \operatorname{tgh} x = 1$	$\lim_{x \to 0^+} \operatorname{ctgh} x = +\infty$
$\lim_{x \to -\infty} \operatorname{senh} x = -\infty$	$\lim_{x \to -\infty} \operatorname{tgh} x = -1$	$\lim_{x \to 0^-} \operatorname{ctgh} x = -\infty$