

# LIMITE DESTRO E SINISTRO

$$\lim_{x \rightarrow c} f(x) = l$$

## LIMITE PER ECCESSO

$$\lim_{x \rightarrow c} f(x) = l^+$$

$\forall \varepsilon > 0 \exists I_\varepsilon^+(l)$  e corrispondente  
mente  $\exists I_\varepsilon^+(c) / \forall x \in I_\varepsilon^+(c)$   
si ha

$$\begin{cases} |f(x) - l| < \varepsilon \\ f(x) \geq l \end{cases} \Leftrightarrow$$

$$0 \leq f(x) - l < \varepsilon$$

## LIMITE DESTRO

$$\lim_{x \rightarrow c^+} f(x) = l$$

$\forall \varepsilon > 0 \exists I_\varepsilon(l)$  e corrispondente  
mente  $\exists I_\varepsilon^+(c) / \forall x \in I_\varepsilon^+(c)$   
si ha

$$\begin{cases} |f(x) - l| < \varepsilon \\ x > c \end{cases}$$

## LIMITE PER DIFETTO

$$\lim_{x \rightarrow c} f(x) = l^-$$

$\forall \varepsilon > 0 \exists I_\varepsilon^-(l)$  e corrispondente  
mente  $\exists I_\varepsilon^-(c) / \forall x \in I_\varepsilon^-(c)$   
si ha:

$$\begin{cases} |f(x) - l| < \varepsilon \\ f(x) \leq l \end{cases} \Leftrightarrow$$

$$-\varepsilon < f(x) - l \leq 0$$

## LIMITE SINISTRO

$$\lim_{x \rightarrow c^-} f(x) = l$$

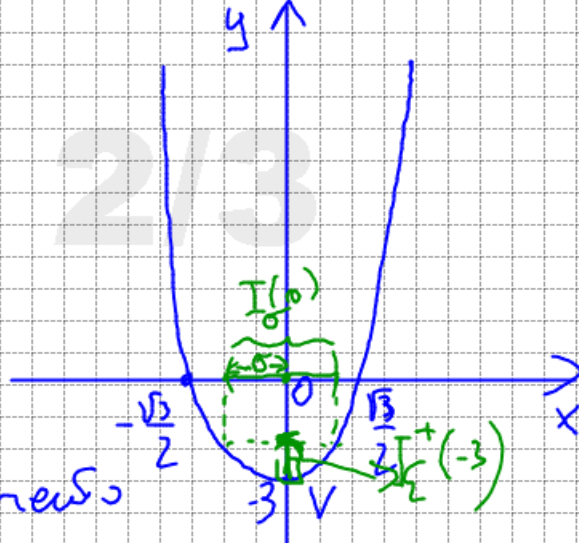
$\forall \varepsilon > 0 \exists I_\varepsilon(l)$  e corrispondente  
mente  $\exists I_\varepsilon^-(c) / \forall x \in I_\varepsilon^-(c)$   
si ha:

$$\begin{cases} |f(x) - l| < \varepsilon \\ x < c \end{cases}$$

# ESEMPIO

$$\lim_{x \rightarrow 0} 4x^2 - 3 = -3^+$$

$$y = 4x^2 - 3$$



$\forall \varepsilon > 0 \exists I^+(-3)$  e contemporaneamente  
 $\exists I_0^+(0) / \forall x \in I_0^+(0)$  si ha che

$$\begin{cases} |f(x) - l| < \varepsilon \\ f(x) \geq l \end{cases} \Leftrightarrow \begin{cases} |4x^2 - 3 + 3| < \varepsilon \\ 4x^2 - 3 \geq -3 \end{cases}$$

$$\begin{cases} 0 \leq 4x^2 < \varepsilon \\ 4x^2 \geq 0 \end{cases} \begin{cases} 4x^2 \geq 0 \text{ ok!} \\ 4x^2 < \varepsilon \\ 4x^2 \geq 0 \text{ ok!} \end{cases}$$

$$0 - \frac{\sqrt{\varepsilon}}{2} < x < +\frac{\sqrt{\varepsilon}}{2} + 0 \quad \bar{e} \text{ un intorno di } 0$$

m 6

$$\lim_{x \rightarrow -1} x^3 - 3 = -4$$

3/3

$\forall \varepsilon > 0 \exists I_\varepsilon(-4)$  e correspondente  $\exists I_\sigma(-1)$

$\forall x \in I_\sigma(-1)$  ai ha de  $|f(x) - l| < \varepsilon \Leftrightarrow$

$$\begin{aligned} |x^3 - 3 + 4| &< \varepsilon \\ |x^3 + 1| &< \varepsilon \end{aligned}$$

$$-\varepsilon < x^3 + 1 < \varepsilon$$

$$\begin{cases} x^3 + 1 + \varepsilon > 0 & x > \sqrt[3]{(-3-\varepsilon-1)} \\ x^3 + 1 - \varepsilon < 0 & x < \sqrt[3]{-3+\varepsilon-1} \end{cases}$$