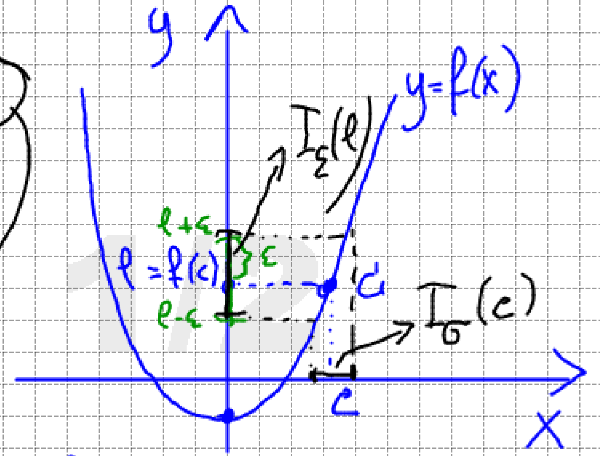
LIMITE FINITO — FINITO

$$\lim_{x \to c} f(x) = \ell$$



Dire che $\lim_{x \to c} f(x) = \ell$ significa dire che:

$\forall \varepsilon > 0 \ \exists \ I_\varepsilon(\ell)$ e corrispondentemente $\exists \ I_\sigma(c)$ t.c.

$\forall x \in I_\sigma(c) \Rightarrow f(x) \in I_\varepsilon(\ell)$ cioè $|f(x) - \ell| < \varepsilon$

$|x - c| < \sigma$

ESEMPIO

$$f(x) = \frac{2x^2 - x - 1}{x - 1} \qquad D_f = \{x \in \mathbb{R} / x - 1 \neq 0\} = (-\infty; 1) \cup (1; +\infty)$$

$$f(1) = ? \quad \frac{2(1)^2 - 1 - 1}{1 - 1} = \frac{0}{0} \ ?$$

$$\lim_{x \to 1} \frac{2x^2 - x - 1}{x - 1} = 3$$ Verifichiamo questo limite:

$\forall \varepsilon > 0 \ \exists \ I_\varepsilon(3)$ e corrispondentemente $\exists$ un intorno di 1 $I_\sigma(1) / \forall x \in I_\sigma(1)$ si ha che $|f(x) - 3| < \varepsilon$

$$\left| \frac{2x^2 - x - 1}{x - 1} - 3 \right| < \varepsilon \qquad \left| \frac{2x^2 - x - 1 - 3x + 3}{x - 1} \right| < \varepsilon$$

$$\left| \frac{2x^2 - 4x + 2}{x - 1} \right| < \varepsilon \qquad -\varepsilon < \frac{2x^2 - 4x + 2}{x - 1} < \varepsilon$$

$$\begin{cases} \dfrac{2x^2 - 4x + 2}{x - 1} > -\varepsilon \\ \dfrac{2x^2 - 4x + 2}{x - 1} < \varepsilon \end{cases} \qquad \begin{cases} \dfrac{2(x-1)^{\cancel{2}}}{\cancel{x-1}} > -\varepsilon \\ \dfrac{2(x-1)^{\cancel{2}}}{\cancel{x-1}} < \varepsilon \end{cases}$$

$$\begin{cases} 2x - 2 + \varepsilon > 0 \\ 2x - 2 - \varepsilon < 0 \end{cases} \qquad \begin{cases} x > \frac{2}{2} - \frac{\varepsilon}{2} \\ x < \frac{2}{2} + \frac{\varepsilon}{2} \end{cases} \qquad \begin{cases} x > 1 - \frac{\varepsilon}{2} \\ x < 1 + \frac{\varepsilon}{2} \end{cases}$$

$$\underbrace{1 - \underbrace{\frac{\varepsilon}{2}}_{\sigma} < x < 1 + \underbrace{\frac{\varepsilon}{2}}_{\sigma}}_{\text{intorno di 1}}$$

$$\lim_{x\to 1} \frac{2x^2-x-1}{x-1} = 0 \qquad \left[\frac{0}{0} = ?\right]$$

$\forall \varepsilon > 0 \ \exists I_\varepsilon(0)$ e corrispondentemente $\exists I_\delta(1) \,/\, \forall x \in I_\delta(1)$ si ha che

$$|f(x) - 0| < \varepsilon \Rightarrow$$

$$\left| \frac{2x^2-x-1}{x-1} - 0 \right| < \varepsilon \qquad \left| \frac{2x^2-x-1}{x-1} \right| < \varepsilon$$

$$x = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \begin{cases} -\frac{1}{2} \\ 1 \end{cases}$$

$ax^2+bx+c=0$

$x_1, x_2 \qquad a(x-x_1)(x-x_2)=0$

$$\left| \frac{2\left(x+\frac{1}{2}\right)\cancel{(x-1)}}{\cancel{(x-1)}} \right| < \varepsilon \qquad -\varepsilon < 2x+1 < \varepsilon$$

$$\begin{cases} 2x+1 > -\varepsilon \\ 2x+1 < \varepsilon \end{cases} \qquad \begin{cases} x > -\frac{1}{2} - \frac{\varepsilon}{2} \\ x < -\frac{1}{2} + \frac{\varepsilon}{2} \end{cases}$$

$$\underbrace{-\frac{1}{2} - \underset{\delta}{\frac{\varepsilon}{2}} < x < -\frac{1}{2} + \underset{\delta}{\frac{\varepsilon}{2}}}_{\text{INTORNO DI } -\frac{1}{2}}$$