DISEQUAZIONI DI II GRADO

- $ax^2+bx+c>0$; $ax^2+bx+c\geq 0$
- $ax^2+bx+c<0$; $ax^2+bx+c\leq 0$

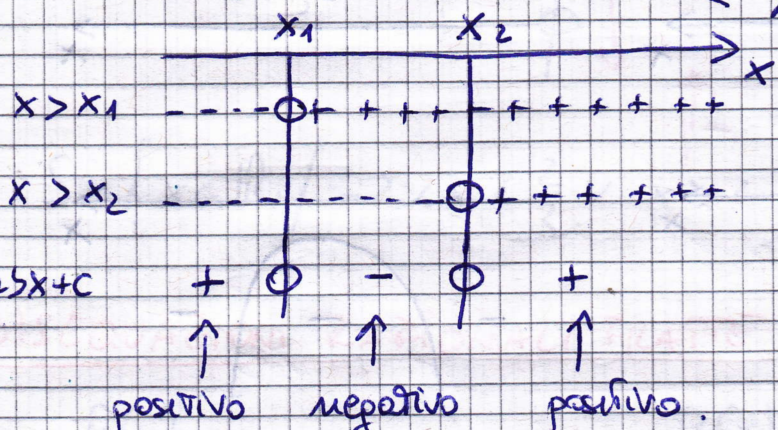
$\boxed{a\neq 0}$

analizziamo i vari casi: 1) $\boxed{\Delta>0 \text{ e } a>0}$

scrivo l'equazione associata e trovo le due radici distinte x_1, x_2 quindi $ax^2+bx+c=a(x-x_1)(x-x_2)$

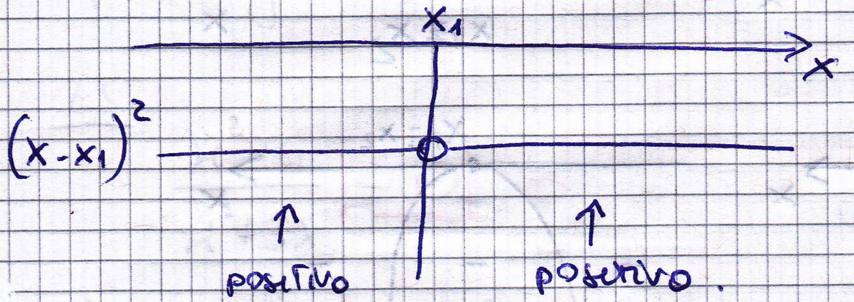
quindi

$$a(x-x_1)(x-x_2)>0 \quad \begin{cases} x>x_1 \\ x>x_2 \end{cases}$$



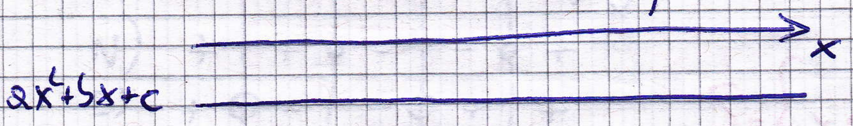
2) $\boxed{\Delta=0 \text{ e } a>0}$ scrivo l'equazione associata e trovo le due radici coincidenti $x_1\equiv x_2$ quindi

$$ax^2+bx+c=a(x-x_1)^2$$



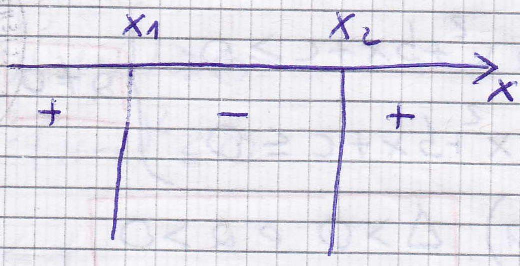
3) $\boxed{\Delta<0 \text{ e } a>0}$

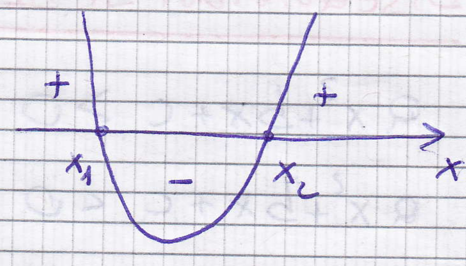
quando $\Delta<0$ le due soluzioni dell'equazione associata non sono reali quindi



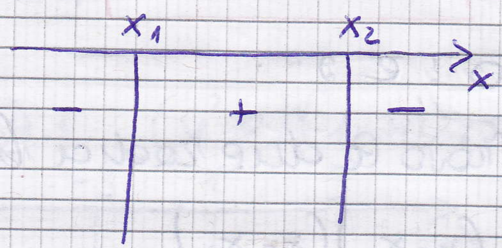
$\boxed{\Delta > 0}$

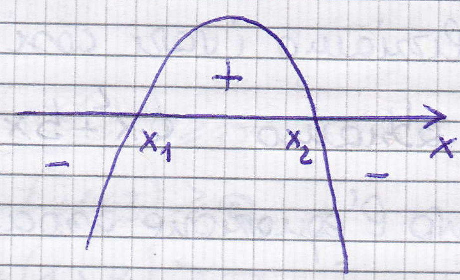
$\boxed{a > 0}$





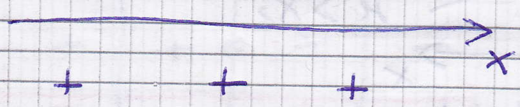
$\boxed{a < 0}$

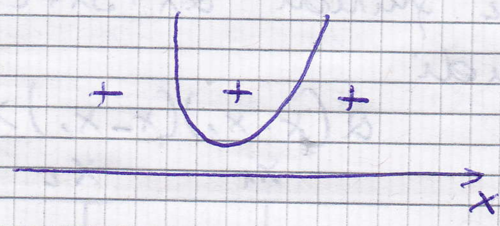




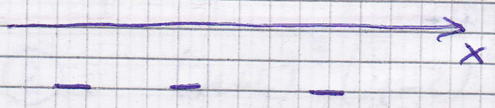
$\boxed{\Delta < 0}$

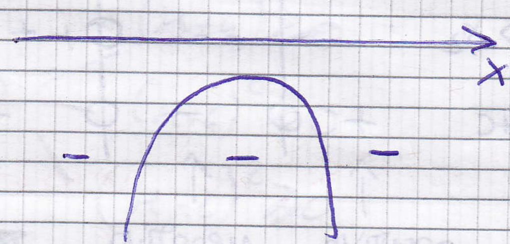
$\boxed{a > 0}$





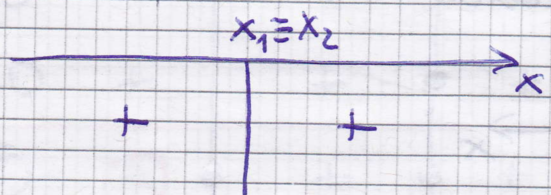
$\boxed{a < 0}$

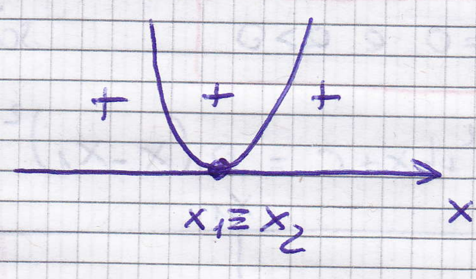




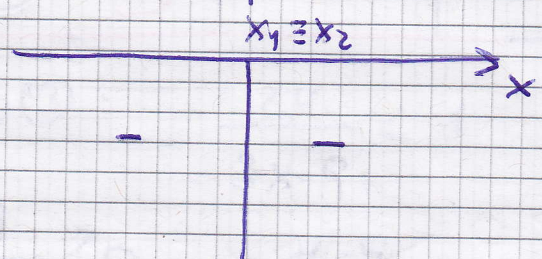
$\boxed{\Delta = 0}$

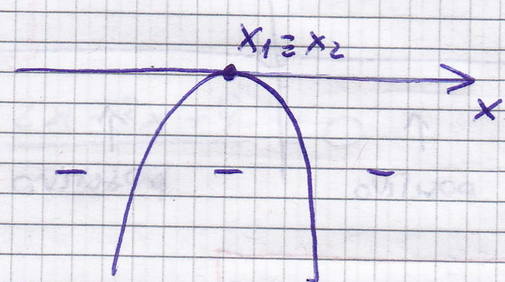
$\boxed{a > 0}$





$\boxed{a < 0}$





DISEQUAZIONI DI GRADO SUPERIORE AL SECONDO

$$(16-x^4)(3x-2)\le 0$$

$$(4-x^2)(4+x^2)(3x-2)\le 0$$

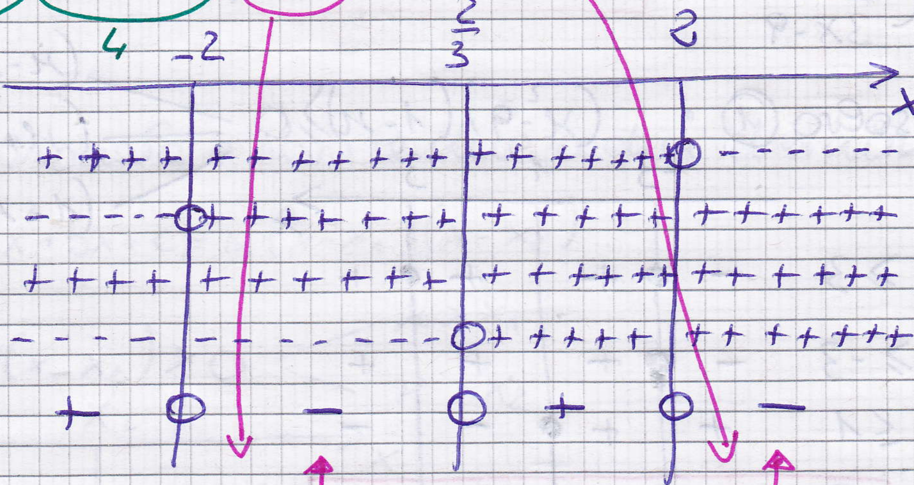
$$\underset{1}{(2-x)}\,\underset{2}{(2+x)}\,\underset{3}{(4+x^2)}\,\underset{4}{(3x-2)}\le 0$$

$2-x>0 \qquad x<2$

$2+x>0 \qquad x>-2$

$4+x^2>0 \qquad$ sempre

$3x-2>0 \qquad x>\frac{2}{3}$



$$S=\left\{x\in\mathbb{R} \,/\, -2<x<\frac{2}{3} \vee x>2\right\}$$

DISEQUAZIONI RAZIONALI FRATTE

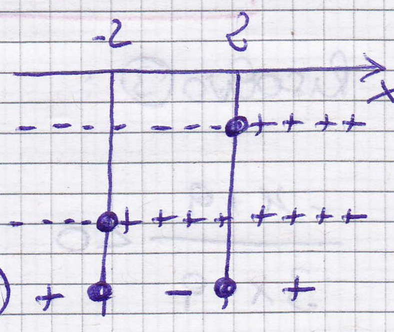
$$\frac{f(x)}{g(x)}>0 \qquad \text{oppure} \qquad \frac{f(x)}{g(x)}<0$$

$f(x)$ e $g(x)$ polinomi

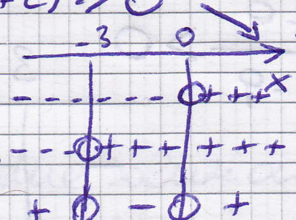
ES

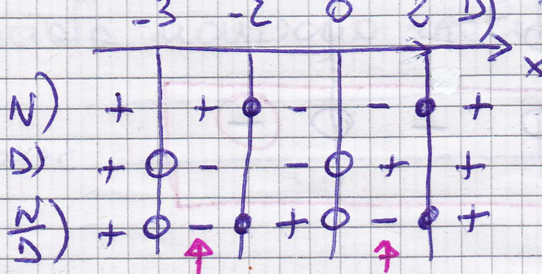
$$\frac{x^2-4}{x^2+3x}\le 0$$

N) $x^2-4\ge 0 \qquad (x-2)(x+2)\ge 0 \to x\ge 2, \; x\ge -2$



D) $x(x+3)>0 \to x>0, \; x>-3$





$$S=\left\{x\in\mathbb{R} \,/\, -3<x\le -2 \vee 0<x\le 2\right\}.$$

SISTEMI DI DISEQUAZIONI

$$\begin{cases} (x^2-9)(1-x) \geq 0 & ① \\ (x+4)^3(x-2)^2 \leq 0 & ② \\ \dfrac{2x}{3x-9} < 1 & ③ \end{cases}$$

Risolvo ①: $(x^2-9)(1-x) \geq 0$

$(x-3) \geq 0$

$(x+3) \geq 0$

$(1-x) \geq 0$

		-3		1		3	
$x \geq 3$	$-$		$-$		$-$	•	$+$
$x \geq -3$	$-$	•	$+$		$+$		$+$
$x \leq 1$	$+$		$+$	•	$-$		$-$
①	$+$	•	$-$	•	$+$	•	$-$

Risolvo ②: $(x+4)^3(x-2)^2 \leq 0$

$(x+4) \geq 0$

$(x+4)^2 \geq 0$

$(x-2)^2 \geq 0$

		-4		2	
$x \geq -4$	$-$	•	$+$		$+$
$(x+4)^2 \geq 0$	$+$	•	$+$		$+$
$(x-2)^2 \geq 0$	$+$		$+$	•	$+$
②	$-$	•	$+$	•	$+$

Risolvo ③: $\dfrac{2x}{3x-9} < 1$ $\qquad \dfrac{2x-3x+9}{3x-9} < 0$

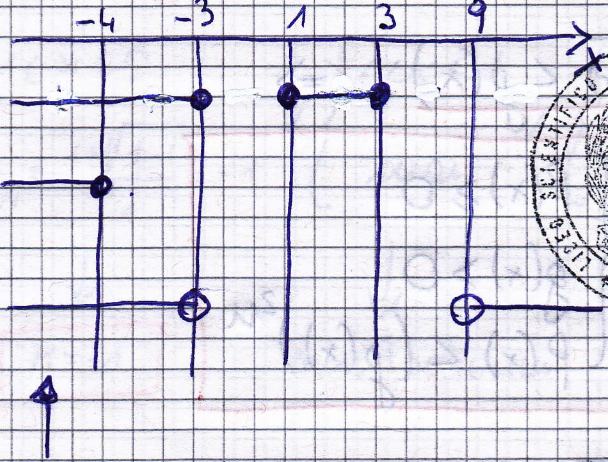
$\dfrac{-x+9}{3x-9} < 0 \qquad \dfrac{x-9}{3x-9} > 0$

N) $x-9>0 \quad x>9$

D) $3x-9>0 \quad x>3$

		3		9	
N	$-$		$-$	∘	$+$
D	$-$	∘	$+$		$+$
$\frac{N}{D}$	$+$	∘	$-$	∘	$+$

$$\begin{cases} x \le -3 \ \vee \ 1 \le x \le 3 & (1) \\ x \le -4 & (2) \\ x < 3 \ \vee \ x > 9 & (3) \end{cases}$$



$$S = \{ x \in \mathbb{R} \ / \ x \le -4 \}.$$

EQUAZIONI E DISEQUAZIONI IRRAZIONALI

$$\boxed{\sqrt[n]{f(x)} = g(x)}$$

n dispari

$$f(x) = [g(x)]^n$$

n pari

$$\begin{cases} f(x) \ge 0 \\ g(x) \ge 0 \\ f(x) = [g(x)]^n \end{cases}$$

$$\boxed{\sqrt[n]{f(x)} \gtrless g(x)}$$

n dispari

$$f(x) \gtrless [g(x)]^n$$

n pari

a) $\sqrt{f(x)} < g(x)$

b) $\sqrt{f(x)} > g(x)$

a) $\sqrt{f(x)} < g(x) \iff$

$$\boxed{\begin{cases} f(x) \geq 0 \\ g(x) > 0 \\ f(x) < (g(x))^2 \end{cases}}$$

b) $\sqrt{f(x)} > g(x) \iff$

$$\boxed{\begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \vee \begin{cases} g(x) \geq 0 \\ f(x) > (g(x))^2 \end{cases}}$$

VALORE ASSOLUTO

$$|a| = \begin{cases} a & \text{se } a \geq 0 \\ -a & \text{se } a < 0 \end{cases}$$

EQUAZIONI CON VALORE ASSOLUTO

$$|x^2-1| + |x| = x+2$$

		-1		0		1		x
$x^2-1 \geq 0$	$+$	●	$-$		$-$	●	$+$	
$x \geq 0$	$-$		$-$	●	$+$		$+$	
	①		②		③		④	

$$\overset{①}{\begin{cases} x < -1 \\ x^2 - 1 - x = x + 2 \end{cases}} \vee \overset{②}{\begin{cases} -1 \leq x < 0 \\ -x^2 + 1 - x = x + 2 \end{cases}} \vee \overset{③}{\begin{cases} 0 \leq x < 1 \\ -x^2 + 1 + \cancel{x} = \cancel{x} + 2 \end{cases}} \vee \overset{④}{\begin{cases} x \geq 1 \\ x^2 - 1 + \cancel{x} = \cancel{x} + 2 \end{cases}}$$

$$\begin{cases} x < -1 \\ x^2 - 2x - 3 = 0 \end{cases} \vee \begin{cases} -1 \leq x < 0 \\ -x^2 - 2x - 1 = 0 \end{cases} \vee \begin{cases} 0 \leq x < 1 \\ -x^2 - 1 = 0 \end{cases} \vee \begin{cases} x \geq 1 \\ x^2 - 3 = 0 \end{cases}$$

$$\begin{cases} x < -1 \\ x_{1,2} = \dfrac{1 \pm \sqrt{1+3}}{1} = 1 \pm 2 \begin{array}{l} \nearrow 3 \\ \searrow -1 \end{array} \end{cases} \vee \begin{cases} -1 \leq x < 0 \\ x_{3,4} = \dfrac{1 \pm \sqrt{1-1}}{-1} = -1 \end{cases} \vee \begin{cases} 0 \leq x < 1 \\ \text{mai} \end{cases} \vee \begin{cases} x \geq 1 \\ x = \pm\sqrt{3} \end{cases}$$

$\downarrow$ non $\exists$ soluzioni

$\downarrow$ $\boxed{x = -1}$

$\downarrow$ non $\exists$ soluzioni

$\downarrow$ $\boxed{x = \sqrt{3}}$

<u>OSS</u>: Discorso analogo per le disequazioni.